On Mean-CVaR Optimization

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Outline

- Non-normal Markets
- Background on VaR
- Shortcomings of VaR
- Introduction to CVaR

Recent Market Shocks

- Latin American debt crisis in the early 1980s
- Stock market crash of 1987
- U.S. Savings & Loan crisis in 1989–1991
- Western European exchange rate mechanism crisis in 1992
- East Asian financial crisis of 1997
- Russian default crisis and the LTCM Hedge Fund crisis in 1998
- Bursting of U.S. Technology bubble in 2000-2001
- Subprime Credit Crisis, 2007-2010
- Greek Debt Crisis, 2010

Serial Correlation

- A central assumption in most asset allocation models is that period returns are independent and identically distributed (by extension, stationary processes).

- However, if one month's return is influenced by the previous month's return, then there is a need to account for clustering and momentum effects.

Fat left tails

- “Fat” Left Tails (Negative Skewness and Leptokurtosis). A second form of non-normality relates to extreme negative events that occur with a considerably higher probability than implied by the normal distribution (approximately 10 times more frequently than expected in the past several decades). This phenomenon is commonly referred to as “fat” left tails.
- More of the variance is due to infrequent extreme events as opposed to frequent modestly sized deviations.

Correlation Breakdown

- Simple correlations are often used in traditional asset allocation models to assume a linear relationship between asset classes. The relationship between the variables at the extremes is similar to their relationship at less extreme values.

- In the empirical results, there is high co-dependence among assets in stress environments- correlations converge as asset prices decline sharply (e.g. forced liquidations and buyer strikes in mortgage derivatives and equity markets in 2008).

Scenario Optimization

- Suppose we obtain a scenario matrix either from historical returns or from a scenario simulation exercise. We could choose a minimax criterion, i.e., we might want to minimize the maximum portfolio loss, referred to as ”minimizing regret”.

- This could be an optimal strategy for investors who have to ensure that under all scenarios they never experience a loss greater than a certain amount. Focusing on extreme events has its merits if returns deviate substantially from normality.

VaR

- In recent years, there has been broad interest in value-at-risk (VaR) as a measure of investment risk. Calculations of VaR help one to assess what maximum loss (either as an absolute monetary value or in terms of return) an investment portfolio might experience with confidence level β over a pre-specified time period.

- We need to specify the level of confidence (typically 0.95 or 95%) as the maximum possible loss will always be 100%.

Shortcomings of VaR

- Interest in VaR as a risk measure are driven by regulatory frameworks, widespread popularity and intuitive appeal. However, there are serious shortcomings:

- Investors subscribing to VaR implicitly state that they are indifferent between very small losses in excess of VaR and very high losses- even total ruin.

Shortcomings of VaR (Part 2)

- VaR is not sub-additive, i.e. we might find portfolio VaR is higher than the sum of individual asset VaRs. Suppose that we invest in a corporate bond with 2% probability of default. With this default probability the VaR at the 95% confidence level is zero.

- However, if we move to a diversified portfolio (with zero default correlation) containing 100 bonds, the probability of at least one default is 87% ($1 – 0.98^{100}$). Hence, portfolio optimization using VaR would result in concentrated portfolios as the VaR measure would indicate the concentrated portfolio was less risky.

Shortcomings of VaR (Part 3)

- VaR has serious mathematical drawbacks. It is a non-smooth, non-convex and multi-extremum (many local minima) function that makes it difficult to use in portfolio construction.

- As VaR-based scenario optimization effectively has to count all losses higher than a moving threshold, keeping track of its changing tail, while at the same time maximizing VaR, is a complicated, mixed integer problem. Little commercially available software is available to overcome this problem.

CVaR

- However, there is a close relative of value-at-risk known as “conditional value at risk” (CVaR). CVaR provides information which is complementary to that given by plain VaR in that it measure expected excess loss over VaR if a loss larger than VaR actually occurs. Hence, it is the average of the worst (1-β) losses.

CVaR (Part 2)

- CVaR must by definition be larger than VAR as it provides information about what kind of losses might hide in the tail of the distribution (hence, its alternative name, “tail VaR”).

- CVaR is a conditional tail expectation- i.e. an expected value for those realizations that fall below the VaR threshold. Also in contrast to VaR, CVaR is sub-additive and can easily be implemented using linear programming, making it computationally very attractive.

References

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